

# **Common Core State Standards (CCSS) for Mathematics 2007 Mississippi Mathematics Framework Revised (MMFR) Alignment Analysis**

## **Mississippi Department of Education Non-Tested Grades**

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**SOUTHEAST COMPREHENSIVE CENTER**

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## Introduction

### Purpose of the Work

The purpose of the alignment analysis of the June 2010, Common Core State Standards (CCSS) for Mathematics to the *2007 Mississippi Mathematics Framework Revised* (MMFR) is to provide guidance to the Mississippi Department of Education (MDE) in the possible implementation of the CCSS for 2011–2012. The analysis will assist MDE in identifying the objectives in the MMFR that will remain or be modified. MDE will conduct a stakeholders meeting in Fall 2010 to obtain input regarding modifications to the existing Mississippi objectives.

### Caveats

To perform this task, the staff of the Southeast Comprehensive Center (SECC) recognized the following situations:

- There are many inherent concepts and skills that are not stated explicitly that students need to have to fulfill the objectives of the MMFR,
- Technical terms may have different names but still may have the same meaning.

To this end, SECC staff included the MMFR objective(s) that best matched the CCSS, even though the alignment was not necessarily a perfect match due to terminology, inclusion, and/or focus.

The notations used in this alignment analysis are taken from both of the documents used, the CCSS for mathematics and the MMFR. Some of the footnotes referenced in the alignment analysis indicate the use of the CCSS glossary and tables that are not included here but are found at the indicated uniform resource locators (URLs) cited in the references.

### Reading the CCSS for Mathematics

Each CCSS for mathematics denotes what students should know and be able to do and are found by grade level Kindergarten through 8 and by conceptual categories for high school. The conceptual categories for high school include: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability.

Within each grade level or conceptual category, standards are found in large groups called domains. For example, 2-MD denotes the grade level two and the domain Measurement and Data. Grade 2 has four domains.

Within each domain are clusters of related standards. For example, the domain of Measurement and Data in grade 2 has four clusters: Measure and estimate lengths in standard units, relate addition and subtraction to length, work with time and money, and represent and interpret data.

### **Reading the 2007 Mississippi Mathematics Framework Revised**

The MMFR is organized by grade level (K–8) and by secondary courses (grades 9–12). The five process standards (problem solving, communication, reasoning and proof, connections, and representation) should permeate all instructional practices. The framework is comprised of five content strands Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

Beneath each content strand are competencies. The competencies are presented in outline form for consistency and for easy reference throughout the framework. Competencies are intentionally broad in order to allow school districts and teachers the flexibility to create a curriculum that meets the needs of their students.

Beneath each competency are objectives. The objectives indicate how competencies can be fulfilled through a progression of content and concepts at each grade level and course. Many of the objectives are interrelated rather than sequential, which means that objectives are not necessarily taught in the specific order in which they are presented. Multiple objectives can and should be taught at the same time.

### **The Format of the Mathematics Alignment Analysis**

This alignment analysis includes the grade levels that are not tested in the state of Mississippi, namely Kindergarten through 2<sup>nd</sup> grade and most of the courses in 9<sup>th</sup> through 12<sup>th</sup>. For matching purposes, the alignment analysis will be done at the CCSS level to the MMFR objective level only.

For each grade level Kindergarten through 2<sup>nd</sup>, SECC staff started with a key describing the numeration systems used by the CCSS and MMFR and then used a three-column table, with the first column containing the CCSS for that grade level, the second column containing the matching MMFR objective, if any for that grade level, and the third column containing comments about observations and other information. The last rows in the table include the MMFR objectives that did not match with any CCSS at that grade level.

For the high school level, SECC staff used a similar process, including a key and a three-column table, but the alignment analysis is not driven by grade level. It is driven by the CCSS conceptual categories of Number and Quantity, Algebra, Functions, Modeling (not a stand-alone conceptual category but embedded in the other five), Geometry, and Statistics and Probability. While there is no true matching between the CCSS conceptual categories and MMFR high school courses, some conceptual categories may align mostly with one MMFR high school course. For example, CCSS Algebra most closely aligns with MMFR Algebra I and is included in the Tested Grades Report.

The high school alignment analysis for the non-tested grades reflects the following pattern:

- CCSS Number and Quantity most closely aligns with MMFR Algebra II but contains some MMFR Trigonometry objectives.
- CCSS Functions goes across the MMFR courses, from Transitions to Algebra to Discrete Mathematics, but contains many Pre-Calculus and Trigonometry objectives.
- CCSS Geometry most closely aligns with MMFR Geometry but contains a few MMFR Trigonometry objectives.

- CCSS Statistics most closely aligns with MMFR Statistics, with several objectives from MMFR Algebra II and Discrete Mathematics.

## References

Mississippi State Department of Education. (2007). *2007 Mississippi mathematics framework revised*. Jackson, MS: Author. Retrieved from <http://www.mde.k12.ms.us/acad/id/curriculum/Math/index.htm>

National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010, June 2). *K–12 common core state standards for mathematics*. Washington, DC: Author. Retrieved from <http://corestandards.org/the-standards/mathematics>

## Grade K

CCSS Key:

Counting and Cardinality (CC)  
 Operations and Algebraic Thinking (OA)  
 Number and Operations in Base Ten (NBT)  
 Measurement and Data (MD)  
 Geometry (G)

MMFR Content Standards Key:

Numbers and Operations (1)  
 Algebra (2)  
 Geometry (3)  
 Measurement (4)  
 Data Analysis and Probability (5)  
 Depth of Knowledge (DOK)

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<p>K.CC.1. Count to 100 by ones and by tens.</p>	<p>K.1.a. Count forward to 20 and backward from 10. (DOK 1)</p>	<p>Counting is included in the MMFR at kindergarten but only by ones to 20 and backward from 10. The MMFR does not specify that students recognize and write numbers to 100 until grade 1.1.a. and that students count by tens until grade 1.2.d.</p>
<p>K.CC.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</p>		<p>The MMFR does not specify that students count forward from a given number other than one; however the MMFR has students explain, analyze, and extend repeating and growing patterns at grade 2.2.a.</p>
<p>K.CC.3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).</p>	<p>K.1.c. Recognize and write numbers to represent quantities 0 to 20. (DOK 1)                      K.1.b. Create models of sets of objects 0 to 20. (DOK 1)                      K.1.d. Compose and decompose two-digit numbers (up to</p>	

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	20) with representations in words and physical models. (DOK 2)	
<p>K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality.</p> <p>a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.</p> <p>b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</p> <p>c. Understand that each successive number name refers to a quantity that is one larger.</p>	<p>K.1.c. Recognize and write numbers to represent quantities 0 to 20. (DOK 1)</p> <p>K.1.b. Create models of sets of objects 0 to 20. (DOK 1)</p> <p>K.1.d. Compose and decompose two-digit numbers (up to 20) with representations in words and physical models. (DOK 2)</p> <p>K.1.e. Determine “first” through “tenth” (ordinal numbers), “next,” and “last” positions. (DOK 1)</p>	<p>The MMFR does not specify that students understand that the last number name tells the number of objects counted or that students understand that each successive number name refers to a quantity that is one larger.</p>
<p>K.CC.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</p>	<p>K.1.b. Create models of sets of objects 0 to 20. (DOK 1)</p> <p>K.1.d. Compose and decompose two-digit numbers (up to 20) with representations in words and physical models. (DOK 2)</p>	<p>The MMFR does not ask students to count to answer “how many?”</p>
<p>K.CC.6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup></p>	<p>K.5.b. Describe data by using mathematical language such as more than, less than, etc. (DOK 1)</p>	<p>The MMFR does not compare or order two-digit numbers using “more”, “less”, “greater than”, “less than”, or “equal to” until grade 1.1.c.</p>
<p>K.CC.7. Compare two numbers between 1 and 10 presented as written numerals.</p>	<p>K.1.g. Apply mathematical language by telling when a certain number is “too many,” “not enough,” “just</p>	<p>The MMFR does not specify that the two numbers being compared are between 1 and 10</p>

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	right,” “more than,” “less than,” or “equal to” for a given situation. (DOK 1)	or that they appear as written numerals.
K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings <sup>2</sup> , sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	K.1.f. Develop multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison). (DOK 2)	
K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	K.1.f. Develop multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison). (DOK 2)	Solving word problems mentioned in the CCSS are not included in the MMFR.
K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$ ).	K.1.d. Compose and decompose two-digit numbers (up to 20) with representations in words and physical models. (DOK 2)	
K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.		The MMFR does not specify that students group numbers in pairs to equal 10. In grade 1.2.c., students solve equations that require addition and subtraction of whole numbers.
<sup>1</sup> Include groups with up to 10 objects.		
<sup>2</sup> Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)		
K.OA.5. Fluently add and subtract within 5.	K.1.f. Develop multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison). (DOK 2)	The MMFR does not specify that students add and subtract within 5 with fluency at grade K. Grade 2.1.a. specifies that students recall addition and subtraction facts.



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<p>K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., <math>18 = 10 + 8</math>); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p>	<p>K.1.d. Compose and decompose two-digit numbers (up to 20) with representations in words and physical models. (DOK 2)</p>	<p>The MMFR does not specify that students understand, when composing and decomposing two-digit numbers up to 20, that the numbers are composed of ten and one, two, three, four, five, six, seven, eight, or nine ones.</p>
<p>K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</p>	<p>K.4.d. Determine attributes of objects that can be compared, such as length, area, mass or volume/capacity. (DOK 1)</p>	
<p>K.MD.2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i></p>	<p>K.4.b. Determine and describe comparisons of length (longer, shorter, the same) mass (heavier, lighter, the same), and capacity (holds more, less, or about the same) using different-shaped or congruent containers, objects or figures. (DOK 2)</p>	
<p>K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.<sup>3</sup></p>	<p>K.2.a. Describe a rule for sorting objects. (DOK 2) K.5.a. Collect and organize data by counting and using tally marks and other symbols. (DOK 1)</p>	
<p><sup>3</sup>Limit category counts to be less than or equal to 10.</p>		
<p>K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to</i>.</p>	<p>K.3.c. Demonstrate an understanding of positional words (e.g., in, above, below, over, under, beside, etc.). (DOK 1)</p>	
<p>K.G.2. Correctly name shapes regardless of their</p>	<p>K.3.b. Identify two-dimensional figures such as the square,</p>	<p>The MMFR does not specify that students correctly name shapes</p>

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orientations or overall size.	rectangle, triangle, and circle. (DOK 1)	regardless of their orientations or size.
K.G.3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).		The MMFR does not specify that students identify shapes as two-dimensional or three-dimensional until grade 1.3.a. and 1.3.b.
K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).		The MMFR does not specify that students identify and classify three-dimensional figures according to their characteristics until grade 2.3.c.
K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.		The MMFR does not specify that students model shapes in the world by building and drawing.
K.G.6. Compose simple shapes to form larger shapes. <i>For example, “Can you join these two triangles with full sides touching to make a rectangle?”</i>		The MMFR at grade 1.3.c. specifies that students explain the part-whole relationships resulting from the composition or decomposition of plane and solid figures.
	K.2.b. Identify, reproduce, and extend repeating patterns in visual, auditory, and physical contexts. (DOK 2)	The CCSS does not specify that students identify patterns until grade 3.
	K.2.c. Identify and describe qualitative changes (such as temperature changes—it feels hotter). (DOK 1)	The CCSS does not identify and describe qualitative changes.

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	K.2.d. Identify and describe quantitative changes (such as temperature increases five degrees). (DOK 1)	The CCSS does not identify and describe quantitative changes.
	K.3.a. Recognize and describe open and closed figures. (DOK 1)	The CCSS does not specify that students recognize and describe open and closed figures.
	K.4.a. Measure the length, weight, and capacity of objects using nonstandard units. (DOK 2)	The CCSS measures length with nonstandard units at grade 1. Weight and capacity are not measured in the CCSS until grade 3.
	K.4.c. Recognize the clock (analog and digital) and calendar measurements of time. (DOK 1)	The CCSS does not specify that students tell time using analog and digital clocks until grade 1. The CCSS does not include the calendar as a measurement of time.

## Grade 1

CCSS Key:

Operations and Algebraic Thinking (OA)  
 Number and Operations in Base Ten (NBT)  
 Measurement and Data (MD)  
 Geometry (G)

MMFR Content Standards Key:

Numbers and Operations (1)  
 Algebra (2)  
 Geometry (3)  
 Measurement (4)  
 Data Analysis and Probability (5)  
 Depth of Knowledge (DOK)

*Note: The first footnote for this grade level is found in the introduction, which is not included here, about the use of "the transitivity principle for indirect measurement."*

<sup>1</sup> Student should apply the principle of transitivity of measurement to make indirect comparisons but they need not to use this technical term.

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1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. <sup>2</sup>	1.1.d Use multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison) to solve problems. (DOK 1) 1.2.b. Formulate, explain, and generalize patterns within and across addition and subtraction. (DOK 2) 1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2)	Solving word problems mentioned in the CCSS is not included in the MMFR.
<sup>2</sup> See Glossary, Table 1.		
1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	1.1.e. Find the sums of 3 single-digit addends (for example: $3 + 6 + 2 = 11$ ). (DOK 1) 1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use	

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<p>1.OA.3. Apply properties of operations as strategies to add and subtract.<sup>3</sup> <i>Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition.) To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.)</i></p>	<p>objects, pictures, and symbols. (DOK 2)</p> <p>1.2.b. Formulate, explain, and generalize patterns within and across addition and subtraction. (DOK 2)</p>	<p>The MMFR does not formally introduce number properties until grade 3.2.c.</p>
<p>1.OA.4. Understand subtraction as an unknown-addend problem. <i>For example, subtract <math>10 - 8</math> by finding the number that makes 10 when added to 8.</i></p>	<p>1.1.d Use multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison) to solve problems. (DOK 1)</p> <p>1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2)</p>	
<p>1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</p>	<p>1.2.d. Count by different units when given a group of objects using 1's, 2's, 5's, and 10's. (DOK 1)</p>	<p>The MMFR does not relate counting to addition and subtraction.</p>
<p><sup>3</sup>Students need not use formal terms for these properties.</p>		
<p>1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., <math>8 + 6 = 8 + 2 + 4 = 10 + 4 = 14</math>); decomposing a number leading to a ten (e.g., <math>13 - 4 = 13 - 3 - 1 = 10 - 1 = 9</math>); using the relationship between addition and subtraction (e.g., knowing that <math>8 + 4 = 12</math>, one knows <math>12 - 8 = 4</math>); and creating equivalent but easier or known sums (e.g., adding <math>6 + 7</math> by creating the known equivalent <math>6 + 6 + 1 = 12 + 1 = 13</math>).</p>	<p>1.1.b. Compose and decompose two-digit numbers with representations in word and physical models. (DOK 2)</p> <p>1.1.d. Use multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison) to solve problems. (DOK 1)</p> <p>1.2.b. Formulate, explain, and generalize patterns within and across addition and subtraction. (DOK 2)</p>	<p>The MMFR does not specify that students use strategies such as counting on and making ten when demonstrating fluency for addition and subtraction.</p>

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<p>1.OA.7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. <i>For example, which of the following equations are true and which are false? <math>6 = 6</math>, <math>7 = 8 - 1</math>, <math>5 + 2 = 2 + 5</math>, <math>4 + 1 = 5 + 2</math>.</i></p>	<p>1.2.c Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2)</p>	<p>The MMFR does not specify that students understand the meaning of the equal sign. The framework does not ask students to determine if equations involving addition and subtraction are true or false.</p>
<p>1.OA.8. Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations <math>8 + ? = 11</math>, <math>5 = \square - 3</math>, <math>6 + 6 = \square</math>.</i></p>	<p>1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2)</p>	
<p>1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p>	<p>1.1.a. Recognize and write numbers 0 to 100. (DOK 1)</p>	<p>The MMFR at kindergarten specifies that students count forward to 20 and backward from 10. The framework does not specify at any other grade level that students “count.”</p>
<p>1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</p> <ul style="list-style-type: none"> <li>a. 10 can be thought of as a bundle of ten ones — called a “ten.”</li> <li>b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</li> <li>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</li> </ul>	<p>1.2.d. Count by different units when given a group of objects using 1’s, 2’s, 5’s, and 10’s. (DOK 1)</p>	<p>The MMFR does not specify that students think of 10 as a bundle of ten ones. Additionally, the framework does not specify that students understand, when composing and decomposing two-digit numbers up to 20, that the numbers are composed of ten and one, two, three, four, five, six, seven, eight, or nine ones.</p>

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<p>1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math>.</p>	<p>1.1.c. Explain how to compare and order two-digit numbers using the terms “more,” “less,” “greater than,” “equal to,” and “almost,” and the symbols <math>&gt;</math>, <math>&lt;</math>, and <math>=</math>. (DOK 1)</p>	<p>The MMFR does not specify that students compare two-digit numbers based on meanings of the tens and ones digits specifically.</p>
<p>1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p>	<p>1.1.d. Use multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison) to solve problems. (DOK 1) 1.1.f. Justify addition and subtraction of two-digit whole numbers without regrouping. (DOK 2) 1.2.b. Formulate, explain, and generalize patterns within and across addition and subtraction. (DOK 2) 1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2)</p>	<p>The MMFR does not specify that students understand that in adding two-digit numbers, sometimes it is necessary to compose a ten.</p>
<p>1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p>	<p>1.2.d. Count by different units when given a group of objects using 1’s, 2’s, 5’s, and 10’s. (DOK 1)</p>	<p>The MMFR grade 1.2.d. objective, counting by 10’s, is a mental skill that would equip a student to be able to mentally find 10 more or 10 less than a number without having to count by ones.</p>
<p>1.NBT.6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition</p>	<p>1.1.d. Use multiple representations for addition (combining of sets) and subtraction (take-away, missing addend, comparison) to solve problems. (DOK 1) 1.1.f.</p>	<p>The MMFR does not specify that students subtract multiples of 10 in the range 10–90; however, the combination of objectives 1.1.d., 1.1.f., 1.2.b., 1.2.c., and 1.2.d. from the framework</p>

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and subtraction; relate the strategy to a written method and explain the reasoning used.	Justify addition and subtraction of two-digit whole numbers without regrouping. (DOK 2) 1.2.b. Formulate, explain, and generalize patterns within and across addition and subtraction. (DOK 2) 1.2.c. Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols. (DOK 2) 1.2.d. Count by different units when given a group of objects using 1's, 2's, 5's, and 10's. (DOK 1)	satisfies the CCSS standard.
1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	1.4.a. Use nonstandard units (paper clips, unifix cubes, etc.) and standard units (inches, centimeters) to measure length. (DOK 1)	The MMFR specifies that students use nonstandard units to measure length; however, the framework does not ask students to order three objects by length by comparing two of the objects indirectly by the third object.
1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</i>	1.4.a. Use nonstandard units (paper clips, unifix cubes, etc.) and standard units (inches, centimeters) to measure length. (DOK 1)	The MMFR does not specify that students understand that the length measurement of an object is the number of same-size length units that span with no gaps or overlaps when measuring with nonstandard units.
1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks.	1.4.d. Tell time to the hour and half-hour intervals using both digital and analog clocks. (DOK 1)	
1.MD.4.	1.5.a.	The MMFR does not specify that



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<p>Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</p>	<p>Gather data, construct, and interpret simple bar graphs and pictographs. (DOK 2) 1.5.b. Analyze and interpret data by using mathematical language such as more than, less than, etc. (DOK 1)</p>	<p>students use up to three categories when organizing, representing, and interpreting data.</p>
<p>1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus nondefining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.</p>	<p>1.3.a. Identify and classify two-dimensional figures (triangle, square, rectangle, circle, trapezoid, hexagon, and rhombus). (DOK 1) 1.3.b. Identify and classify three-dimensional figures (cube, rectangular prism, and sphere) according to their characteristics. (DOK 1)</p>	<p>The MMFR does not specify that students distinguish between defining attributes versus nondefining attributes, nor does the framework have students to build and draw shapes possessing defining attributes.</p>
<p>1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.<sup>4</sup></p>	<p>1.3.c. Explain the part-whole relationships resulting from the composition or decomposition of plane and solid figures. (DOK 2)</p>	<p>In composing and decomposing two- and three-dimensional shapes, the MMFR does not specify that students compose a new shape from the composite shape; however, at grade 2.3.b., students describe the effects of composition and decomposition of polygons when smaller shapes are substituted for a larger shape or a larger shape is substituted for smaller ones.</p>
<p>1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.</p>		<p>The MMFR does not specify that students identify and model representations of fractions (halves, thirds, fourths, fifths, sixths, and eighths) until grade 3.1.d., nor does the framework ask that students describe the partition as two of or four of the</p>

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		shares or understand that decomposing into more equal shares creates smaller shares.
	1.1.a. Recognize and write numbers 0 to 100. (DOK 1)	The CCSS K.CC.1. standard specifies that students count to 100 by ones and by tens in kindergarten.
	1.1.g. Find equal money amounts with different coin combinations up to \$0.25. (DOK 1) 1.1.h. Identify the value of like coins up to \$1.00. (DOK 1) 1.1.i. Determine the value of like coins up to \$1.00. (DOK 1) 1.1.j. Find the value of mixed coins up to \$1.00. (DOK 1)	The CCSS does not specify that students work with money problems until grade 2 at standard 2.MD.8., "Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i> "
	1.2.a. Use a pattern rule to translate and recognize patterns from one pattern representation to another. (DOK 2)	The CCSS introduces generalizing and analyzing patterns at grade 3.OA.9., "Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i> "
	1.4.b. Compare weight of objects using a balance scale with and without nonstandard units. (DOK 1)	The CCSS does not specify that students solve problems involving measurement of liquid

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	1.4.c. Compare and estimate capacity of various containers in nonstandard units. (DOK 2)	volumes and masses of objects until grade 3.
<sup>4</sup> Students do not need to learn formal names such as “right rectangular prism.”		

## Grade 2

CCSS Key:

Operations and Algebraic Thinking (OA)  
 Number and Operations in Base Ten (NBT)  
 Measurement and Data (MD)  
 Geometry (G)

MMFR Content Standards Key:

Numbers and Operations (1)  
 Algebra (2)  
 Geometry (3)  
 Measurement (4)  
 Data Analysis and Probability (5)  
 Depth of Knowledge (DOK)

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2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. <sup>1</sup>	2.1.b. Justify addition and subtraction of two- and three-digit whole numbers with and without regrouping. (DOK 2) 2.2.c. Model situations and solve equations that involve the addition and subtraction of whole numbers. (DOK 2)	Solving word problems mentioned in the CCSS is not included in the MMFR. Additionally, the MMFR does not specify that students solve <i>two-step</i> word problems or that a symbol be used for the unknown to represent the problem.
2.OA.2. Fluently add and subtract within 20 using mental strategies. <sup>2</sup> By end of Grade 2, know from memory all sums of two one-digit numbers.	2.1.a. Recall addition and subtraction facts. (DOK 1)	The MMFR does not specify that students fluently add and subtract <i>within 20</i> .
<sup>1</sup> See Glossary, Table 1.		
<sup>2</sup> See standard 1.OA.6 for a list of mental strategies.		
2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	2.2.b. Use number patterns to skip count by 2's, 3's, 5's, and 10's. (DOK 1)	The MMFR does not specify that students determine whether a group of objects has an odd or even number of members or that students express an even number as a sum of two equal addends.
2.OA.4. Use addition to find the total number of objects	2.2.b. Use number patterns to skip count by 2's, 3's, 5's,	The MMFR does not specify that students use addition to find the

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arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	and 10's. (DOK 1) 2.2.c. Model situations and solve equations that involve the addition and subtraction of whole numbers. (DOK 2)	total number of objects arranged in rectangular arrays; however, MS objective 2.2.c. specifies that students model situations that involve addition of whole numbers, which could include arrays.
2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:  a. 100 can be thought of as a bundle of ten tens—called a “hundred.”  b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	2.1.c. Compose and decompose three-digit numbers with representations in words and physical models. (DOK 2)	The MMFR does not specify that students, when composing and decomposing three-digit numbers, represent amounts of hundreds, tens, and ones. Additionally, the framework does not specify that students understand that 100 can be thought of as a bundle of ten tens or that the numbers 100 through 900 refer to a number of hundreds.
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.	2.2.b. Use number patterns to skip count by 2's, 3's, 5's, and 10's. (DOK 1)	The MMFR does not specify that when skip counting, students skip count by 100's or that when skip counting by 2's, 3's, 5's, and 10's, they do so within 1000.
2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.		The MMFR does not specify that students read and write four-digit whole numbers using number names, expanded form, and standard form until grade 3.1.a.
2.NBT.4.	2.1.e.	

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<p>Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p>Compare and order three-digit numbers using the symbols <math>&lt;</math>, <math>&gt;</math>, and <math>=</math>, and justify reasoning. (DOK 1)</p>	
<p>2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>2.1.b. Justify addition and subtraction of two- and three-digit whole numbers with and without regrouping. (DOK 2) 2.2.c. Model situations and solve equations that involve the addition and subtraction of whole numbers. (DOK 2) 2.2.d. Analyze and generalize the inverse relationships between addition and subtraction. (DOK 2)</p>	<p>The MMFR does not introduce properties of operations until grade 3.2.c. The MMFR does not specify that students use strategies based on place value when adding and subtracting within 100.</p>
<p>2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p>	<p>2.1.b. Justify addition and subtraction of two- and three-digit whole numbers with and without regrouping. (DOK 2)</p>	<p>The MMFR does not specify how many two-digit numbers students are to add. The MMFR does not introduce properties of operations until grade 3.2.c.</p>
<p>2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p>	<p>2.1.b. Justify addition and subtraction of two- and three-digit whole numbers with and without regrouping. (DOK 2) 2.1.c. Compose and decompose three-digit numbers with representations in words and physical models. (DOK 2) 2.2.c. Model situations and solve equations that involve the addition and subtraction of whole numbers. (DOK 2) 2.2.d. Analyze and generalize the inverse relationships</p>	<p>The MMFR does not introduce properties of operations until grade 3.2.c.</p>

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	between addition and subtraction. (DOK 2)	
<p>2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p>	<p>2.2.b. Use number patterns to skip count by 2’s, 3’s, 5’s, and 10’s. (DOK 1)</p>	<p>While MMFR objective 2.2.b. specifies that students skip count by 10’s, it does not require that students mentally add or subtract by 10 to a given number 100–900 or that students skip count or mentally add and subtract by 100 to a given number 100–900.</p>
<p>2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.<sup>3</sup></p>		<p>The MMFR does not specify that students explain why addition and subtraction strategies work using place value. The MMFR does not introduce properties of operations until grade 3.2.c.</p>
<p><sup>3</sup>Explanations may be supported by drawings or objects.</p>		
<p>2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p>	<p>2.4.a. Select appropriate tools and units, estimate, and measure length (to the nearest inch, foot, yard, centimeter, and meter), capacity (to the nearest ounce, cup, pint, quart, gallon, and liter), and weight (to the nearest ounce, pound, gram, and kilogram). (DOK 2)</p>	
<p>2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.</p>		<p>The MMFR does not specify that students measure the length of an object twice, using length units of different lengths.</p>
<p>2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.</p>	<p>2.4.a. Select appropriate tools and units, estimate, and measure length (to the nearest inch, foot, yard, centimeter, and meter), capacity (to the nearest</p>	

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	ounce, cup, pint, quart, gallon, and liter), and weight (to the nearest ounce, pound, gram, and kilogram). (DOK 2)	
2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.		The MMFR does not specify that students measure to determine how much longer one object is than another.
2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.		Using addition and subtraction within 100 to solve word problems involving lengths that are given in the same units mentioned in the CCSS is not included in the MMFR.
2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.		The MMFR does not specify that students represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2,..., or that students represent whole number sums and differences within 100 on a number line.
2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	2.4.b. Read and write time to the hour, half-hour, quarter-hour, and five-minute intervals using digital and analog clocks. (DOK 1)	
2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i>	2.1.f. Determine and compare the value of money up to \$5.00 using the appropriate symbols for dollars and cents. (DOK 1)	Solving word problems mentioned in the CCSS is not included in the MMFR.



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<p>2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p>	<p>2.5.a. Tally, record, interpret, and predict outcomes based on given information. (DOK 3) 2.5.b. Create line graphs, bar graphs, and pictographs using real data. (DOK 2)</p>	<p>The MMFR does not specify that students generate measurement data by measuring lengths of several objects. Objective 2.5.b. includes creating line graphs, which could be made from measurement data.</p>
<p>2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems<sup>4</sup> using information presented in a bar graph.</p>	<p>2.5.b. Create line graphs, bar graphs, and pictographs using real data. (DOK 2)</p>	<p>The MMFR objective 2.5.b. does not specify that students create graphs using a data set with up to four categories. Additionally, the MMFR does not specify that students compare data, interpret quantities, make predictions, or solve problems based on the information represented on tables and different types of graphs (line graphs, pictographs, and bar graphs) until grade 3.5.a.</p>
<p><sup>4</sup>See Glossary, Table 1.</p>		
<p>2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.<sup>5</sup> Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.</p>	<p>2.3.a. Recognize and identify polygons (rhombus, square, triangle, trapezoid, rectangle, pentagon, hexagon, octagon, and decagon) according to the number of sides. (DOK 1) 2.3.c. Identify and classify three-dimensional figures (cone, pyramid, and cylinder) according to their characteristics. (DOK 1)</p>	<p>MMFR objectives 2.3.a. and 2.3.c. do not specify that students draw shapes having specified attributes.</p>
<p>2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p>	<p>2.3.b. Describe the effects of composition and decomposition of polygons when smaller shapes are substituted for a larger shape or a larger shape</p>	<p>MMFR objective 2.3.b. does not specify that when students decompose polygons into smaller shapes they count to</p>

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	is substituted for smaller ones. (DOK 2)	find the total number of them.
<sup>5</sup> Sizes are compared directly or visually, not compared by measuring.		
<p>2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p>	<p>2.3.b. Describe the effects of composition and decomposition of polygons when smaller shapes are substituted for a larger shape or a larger shape is substituted for smaller ones. (DOK 2)</p>	<p>The MMFR does not specify that students identify and model representations of fractions (halves, thirds, fourths, fifths, sixths, and eighths) until grade 3.1.d. The MMFR does not specify that students describe equal shares of the whole as <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc. Additionally, the MMFR does not specify that students recognize that equal shares of identical wholes need not have the same shape.</p>
	<p>2.1.d. Round up to three-digit whole numbers to the nearest hundreds. (DOK 1)</p>	<p>The CCSS do not include standards for rounding whole numbers.</p>
	<p>2.2.a. Explain, analyze, and extend repeating and growing patterns. (DOK 2)</p>	<p>The CCSS do not introduce arithmetic patterns until grade 3.</p>

## Number and Quantity/Algebra II (All)

CCSS Key:

- The Real Number System (N-RN)
- Quantities (N-Q)
- The Complex Number System (N-CN)
- Vector and Matrix Quantities (N-VM)
- Modeling Standards
- (+) Standards that Include Complex Numbers on Complex Plane in Rectangular and Polar Form (Includes Real and Imaginary Numbers)

MMFR Content Standards Key:

- Numbers and Operations (1)
- Algebra (2)
- Geometry (3)
- Measurement (4)
- Data Analysis & Probability (5)
- Depth of Knowledge (DOK)

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N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3)3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i>	All.1.b. Compute with rational and radical expressions and complex numbers, expressing in simplest form. (DOK 1)	The CCSS calls for explanation of the definition of rational exponents. The set of MMFR objectives is more about computing and writing equivalent forms of rational expressions and solving equations with rational expressions.
N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	All.1.b. Compute with rational and radical expressions and complex numbers, expressing in simplest form. (DOK 1)  All.2.g. Solve radical equations. (DOK 2)  All.2.i. Solve equations involving rational expressions and verify solutions. (DOK 2)	Inherent in computing with rational and radical expressions and solving radical equations is using the properties of exponents, which is not explicitly stated in the MMFR.
N-RN.3.		While inherent in the study of rational

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<p>Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</p>		<p>and irrational numbers in Pre-Algebra and Geometry MMFR, this CCSS as stated is not found in the MMFR.</p>
<p>N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p>	<p>All.4.a. Verify the appropriateness of the numerical value and the units of a variable in an equation. (DOK 2)</p>	<p>While inherent in the study of graphing and measurement in the MMFR across all grade levels, the MMFR does not focus on the use of units as a way to guide and understand solutions for multistep problems.</p>
<p>N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.</p>	<p>All.5.c. Model a data set using the median-fit-method with a linear equation and make predictions based on the model and the equation. (DOK 3)</p>	<p>This MMFR objective is more on prescriptive modeling than descriptive modeling. The term descriptive modeling is not found in the MMFR.</p>
<p>N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>	<p>All.4.b. Describe the level of accuracy of measurements in real-world situations by using absolute value inequalities. (DOK 1)</p>	<p>Both the CCSS and the 4.b MMFR objective address level of accuracy of measurements, but the MMFR objective focuses on using absolute value inequalities.</p>
<p>N-CN.1. Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p>	<p>All.1.a. Diagram the relationship among the subsets of the complex number system. (DOK 2)</p> <p>All.1.c. Evaluate powers of the imaginary unit, <math>i</math>. (DOK 1)</p>	<p>The focus of CCSS is in knowing the form of a complex number while the MMFR objective calls for diagramming relationships among the subsets of the complex number system.</p>
<p>N-CN.2. Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p>	<p>All.1.b. Compute with rational and radical expressions and complex numbers, expressing in simplest form. (DOK 1)</p>	<p>The MMFR includes computing with rational and radical expressions in addition to computing with complex numbers.</p>

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<p>N-CN.3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p>	<p>All.2.h. Write equivalent forms of rational expressions using real and complex conjugates. (DOK 2)</p>	<p>The CCSS specifies using conjugates to find moduli and quotients of complex numbers.</p>
<p>N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p>	<p>All.3.d. Represent complex numbers and the sum of complex numbers in a complex coordinate plane. (DOK 1)</p>	<p>While the MMFR objective and the CCSS call for representing complex numbers in a complex coordinate plane, the CCSS includes representation in polar form.</p>
<p>N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, <math>(1 - \sqrt{3}i)^3 = 8</math> because <math>(1 - \sqrt{3}i)</math> has modulus 2 and argument <math>120^\circ</math>.</i></p>	<p>All.3.d. Represent complex numbers and the sum of complex numbers in a complex coordinate plane. (DOK 1)</p> <p>All.1.c. Evaluate powers of the imaginary unit, <math>i</math>. (DOK 1)</p>	<p>The CCSS includes other operations of complex numbers besides sums.</p> <p>Inherent in representing operations with complex numbers is using powers of the imaginary unit, <math>i</math>, which is not stated in this CCSS but probably alluded to in CCSS N-CN.1.</p>
<p>N-CN.6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</p>		<p>This CCSS is not found in the MMFR with this type of specificity.</p>
<p>N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.</p>	<p>All.2.b. Solve systems of absolute value and quadratic equations using a variety of solution methods including graphing. (DOK 2)</p> <p>All.2.d. Given the solution(s) to a quadratic equation, find a quadratic equation to fit the solution(s)</p>	<p>These MMFR objectives involving working with quadratic equations but solving quadratic equations with real coefficients that have complex solutions are not explicitly stated.</p>

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	<p>and explain or justify the solution process. (DOK 2)</p> <p>All.2.e. Use the discriminant to classify and predict the types of solutions of quadratic equations and justify the classification. (DOK 2)</p> <p>All.2.i. Interpret the zeros and maximum or minimum value(s) of quadratic functions. (DOK 2)</p>	
<p>N-CN.8. (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i></p>	<p>All.1.b. Compute with rational and radical expressions and complex numbers, expressing in simplest form. (DOK 1)</p>	<p>CCSS describes much more specific expectations.</p>
<p>N-CN.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>		<p>The Fundamental Theorem of Algebra is not included in the MMFR objectives.</p>
<p>N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., <math>\mathbf{v}</math>, <math> \mathbf{v} </math>, <math>\ \mathbf{v}\ </math>, <math>v</math>).</p> <p>N-VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p>		<p>Recognizing and drawing different notations for vectors to represent a quantity is in Trigonometry in the MMFR.</p>

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<p>N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p>N-VM.4. (+) Add and subtract vectors.</p> <p>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p>c. Understand vector subtraction <math>\mathbf{v} - \mathbf{w}</math> as <math>\mathbf{v} + (-\mathbf{w})</math>, where <math>-\mathbf{w}</math> is the additive inverse of <math>\mathbf{w}</math>, with the same magnitude as <math>\mathbf{w}</math> and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p>		<p>Analyzing properties of vectors and their effects on operations with vectors begins in Trigonometry in the MMFR.</p>
<p>N-VM.5. (+) Multiply a vector by a scalar.</p> <p>a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as <math>c(v_x, v_y) = (cv_x, cv_y)</math>.</p> <p>b. Compute the magnitude of a scalar multiple <math>c\mathbf{v}</math> using <math>\ c\mathbf{v}\  =  c \mathbf{v}</math>. Compute the direction of <math>c\mathbf{v}</math> knowing that when <math> c \mathbf{v} \neq 0</math>, the direction of <math>c\mathbf{v}</math> is either along <math>\mathbf{v}</math> (for <math>c &gt; 0</math>) or against <math>\mathbf{v}</math> (for <math>c &lt; 0</math>).</p>		<p>Analyzing properties of vectors and their effects on operations with vectors begins in Trigonometry in the MMFR.</p>

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<p>N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p>		<p>Using matrices to solve these types of mathematical situations and contextual problems is found in Discrete Mathematics in the MMFR.</p>
<p>N-VM.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</p> <p>N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.</p> <p>N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</p> <p>N-VM.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</p> <p>N-VM.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</p>	<p>All.1.d. Perform computations, including addition, scalar multiplication, multiplication, determinants, and inverses on matrices. (DOK 1)</p>	<p>The MMFR objective is all-inclusive, incorporating computations involving addition, scalar multiplication, multiplication, determinants, and inverses on matrices, while the CCSS N-VM.7 through N-VM.10 focus on specific operations and properties of matrices. The MMFR does not address matrices as transformations of vectors.</p>
<p>N-VM.12. (+) Work with <math>2 \times 2</math> matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.</p>	<p>All.1.d. Perform computations, including addition, scalar multiplication, multiplication, determinants, and inverses on matrices.</p>	<p>The MMFR objective is all-inclusive, incorporating computations involving addition, scalar multiplication, multiplication, determinants, and</p>



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	(DOK 1)	inverses on matrices, but does not necessarily address matrices as transformations of the plane.
	All.1.f. Explain and use the inverse relationship between exponential and logarithmic expressions. (DOK 2)	Understanding the inverse relationship between exponents and logarithms is found in Functions CCSS.
	All.1.g. Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values. (DOK 1)	Considering cases with logarithmic functions is in two categories in the CCSS: Algebra and Functions.
	All.1.h. Solve application problems involving exponential functions related to growth and decay. (DOK 3)	Using properties of exponents to interpret expressions for exponential functions and classifying them as representing exponential growth or decay is in Functions CCSS.
	All.2.a. Solve compound and absolute value inequalities, graphing and writing solutions in interval notation. (DOK 2)	Working with compound and absolute value inequalities begins in grade 7 and continues in Algebra (A-REI) and Functions (F-IF) of the CCSS.
	All.2.c. Given constraints, find the maximum and minimum value(s) of a system of linear inequalities and explain your reasoning. (DOK 2)	Linear inequalities are found in Algebra (A-REI) of the CCSS.
	All.2.f. Factor sums and differences of cubes and factor polynomials by grouping. (DOK 2)	Factoring polynomials is found in Algebra (A-SSE) of the CCSS.
	All.2.j. Explain the results of compositions of functions.	While the CCSS include functions beginning in 8 <sup>th</sup> grade, composition of

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	(DOK 2)	functions is in Functions (F-BF) of the CCSS.
	All.2.k. Explain the Binomial Theorem and use it to expand binomial expressions raised to positive integral powers. (DOK 2)	The Binomial Theorem is in Algebra (A-APR) of the CCSS.
	All.3.a. Determine and justify whether the inverse of a relation or a function exists. (DOK 2)	Inverse functions are included in Functions, CCSS.
	All.3.b. Classify functions based on sketches of their graphs. (DOK 2)	Using functions to model relationships between quantities is found in 8 <sup>th</sup> grade CCSS.
	All.3.c. Sketch and describe transformations of quadratic and absolute value functions. (DOK 2)	Describing transformations as functions is in Geometry, CCSS.
	All.3.e. Identify and sketch the essential graphs of the four conic sections: circle, parabola, ellipse, and hyperbola. (DOK 1)	Translating between the geometric description and the equation for a conic section is in Geometry CCSS.
	All.5.a. Through the use of technology, use scatter plots and linear and quadratic regression analysis to determine an appropriate function to model real-life data. (DOK 3)	The importance of technology to generate plots and regression functions is found in Statistics and Probability, CCSS.
	All.5.b. Solve simple combinations. (DOK 2)	Permutations and combinations are found in Statistics and Probability CCSS.

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	All.5.d. Identify the difference between permutations and combinations and use them to solve real-world problems. (DOK 2)	Permutations and combinations are found in Statistics and Probability, CCSS.

## Functions

CCSS Key:

- Interpreting Functions (F-IF)
- Building Functions (F-BF)
- Linear and Exponential Models  (F-LE)
- Trigonometric Functions (F-TF)
- Modeling Standards
- (+) Standards that Include Complex Numbers on Complex Plane In Rectangular and Polar Form (Includes Real and Imaginary Numbers)

MMFR Content Standards Key:

- Numbers and Operations (1)
- Algebra (2)
- Geometry (3)
- Measurement (4)
- Data Analysis & Probability (5)
- Depth of Knowledge (DOK)
- Geometric Modeling

**Note:** Objectives about functions transcend the grade levels in the MMFR. For this alignment analysis, there will be references in the middle column to the mathematics courses in the MMFR and the objectives that are matched to the CCSS. Courses in the MMFR include Pre-Algebra (PA), Transitions to Algebra (TA), Algebra I (AI), Algebra II (AII), Advanced Algebra (AA), Pre-Calculus (PC), Calculus (C), Trigonometry (T), and Discrete Mathematics (DM). Because the fourth geometric strand of the Trigonometry course has specific references to geometric modeling, those objectives are marked with . The last rows in the table below include MMFR objectives about functions not found in the CCSS.

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<p>F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p>		<p>The idea of function begins in 4<sup>th</sup> grade and is included in subsequent grades through 7<sup>th</sup> grade MMFR. It is unclear where students begin using functional notation but by Algebra I MMFR, students are expected to analyze the relationship between two quantities to determine whether the relation is a function, identifying domain and range.</p>
<p>F-IF.3. Recognize that sequences are functions, sometimes</p>	<p>PC.1.a. Express sequences and series using recursive and</p>	<p>The MMFR does not overtly connect sequences with the</p>

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<p>defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>	<p>explicit formulas. (DOK 2)</p> <p>DM.2.e. Define a sequence recursively and explicitly. (DOK 2)</p> <p>PC.2.f. Find the explicit formula for a recursively-defined sequence using iteration. (DOK 2)</p>	<p>idea of functions.</p>
<p>F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>□</p>	<p>TA.2.g. Identify domain, range, slope, and intercepts of functions. (DOK 1)</p> <p>All.3.b Classify functions based on sketches of their graphs. (DOK 2)</p> <p>C.2.a. Predict and explain the characteristics and behavior of functions and their graphs (domain, range, increasing/decreasing intervals, intercepts, symmetry, and end behavior). (DOK 2)</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>
<p>F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>□</p>	<p>TA.2.g. Identify domain, range, slope, and intercepts of functions. (DOK 1)</p> <p>Al.2.c. Analyze the relationship between <math>x</math> and <math>y</math> values, determine whether a relation is a function, and identify domain and range. (DOK 2)</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>
<p>F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from</p>	<p>C.7.a. Demonstrate and explain the differences between average and instantaneous rates of change. (DOK 2)</p>	

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a graph.□		
<p>F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.□</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>	<p>PC.2.a Determine characteristics of graphs of parent functions (domain/range, increasing/decreasing intervals, intercepts, symmetry, end behavior, and asymptotic behavior). (DOK 2)</p> <p>PC.3.a. Describe the attributes of graphs and the general equations of parent functions (linear, quadratic, cubic, absolute value, rational, exponential, logarithmic, square root, cube root, and greatest integer). (DOK 1)</p> <p>AI.2.e. Graph and analyze linear functions. (DOK 2)</p> <p>AI.2.l. Interpret the zeros and maximum or minimum value(s) of quadratic functions. (DOK 2)</p> <p>AI.2.k. Graph and analyze absolute value and quadratic functions. (DOK 2)</p> <p>PC.2.c. Determine the domain and range of functions, including piece-wise functions. (DOK 2)</p> <p>PC.3.c. Predict the shapes of graphs of exponential, logarithmic, rational, and piece-wise functions, and verify the prediction with and without technology. (DOK 2)</p>	<p>There are several correlations, but they are scattered across higher level courses in the MMFR, primarily at the Pre-Calculus level.</p>

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<p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<p>PC.2.d. Determine the end behavior of polynomial functions. (DOK 2)</p> <p>PC.2.h. Find the zeros of polynomial functions by synthetic division and the Factor Theorem. (DOK 1)</p> <p>PC.2.b. Determine horizontal, vertical, and slant asymptotes and holes of rational functions and explain how each was found. (DOK 2)</p> <p>C.2.b. Investigate, describe, and determine asymptotic behavior using tables, graphs, and analytical methods. (DOK 2)</p> <p>PC.2.j. Decompose a rational function into partial fractions. (DOK 2)</p> <p>PC.2.f. Solve exponential and logarithmic equations to include real-world applications. (DOK 2)</p> <p>T.4.d. Recognize, sketch, and interpret graphs of the six trigonometric functions and include restrictions on their domain. (DOK 2)</p>	
<p>F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the</p>	<p>Al.2.i.</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>

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<p>square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</p>	<p>Determine the solutions to quadratic equations by using graphing, tables, completing the square, the Quadratic formula, and factoring. (DOK 1)</p> <p>All.2.l. Interpret the zeros and maximum or minimum value(s) of quadratic functions. (DOK 2)</p> <p>AA.1.e. Solve application problems involving <math>e</math> and exponential functions related to growth and decay. (DOK 3)</p> <p>All.1.h. Solve application problems involving exponential functions related to growth and decay. (DOK 3)</p>	
<p>F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>	<p>Al.2.k. Graph and analyze absolute value and quadratic functions. (DOK 2)</p> <p>All.3.e. Identify and sketch the essential graphs of the four conic sections: circle, parabola, ellipse, and hyperbola. (DOK 1)</p>	<p>Specific correlations exist only by liberal interpretation of more generic MMFR objectives.</p>
<p>F-BF.1. Write a function that describes a relationship between two quantities. □</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying</p>	<p>PC.1.a. Express sequences and series using recursive and explicit formulas. (DOK 2)</p> <p>DM.2.e. Define a sequence recursively and explicitly. (DOK 2)</p> <p>AA.2.a. Find the sum, difference, product, and quotient of functions, noting any restrictions on the domain.</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>



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<p>exponential, and relate these functions to the model.</p> <p>c. (+) Compose functions. For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</p>	<p>(DOK 2)</p> <p>All.2.j. Explain the results of compositions of functions. (DOK 2)</p> <p>PC.2.e. Decompose composite functions into component functions. (DOK 2)</p>	
<p>F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.□</p>	<p>PC.1.a. Express sequences and series using recursive and explicit formulas. (DOK 2)</p> <p>PC.1.b. Evaluate and apply formulas for arithmetic and geometric sequences and series. (DOK 2)</p>	
<p>F-BF.3. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>PC.3.b. Explain the effects of changing the parameters in transformations of functions. (DOK 2)</p> <p>All.3.c. Sketch and describe transformations of quadratic and absolute value functions. (DOK 2)</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>
<p>F-BF.4. Find inverse functions.</p> <p>a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> for <math>x &gt; 0</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></p> <p>b. (+) Verify by composition that one function is the</p>	<p>All.3.a. Determine and justify whether the inverse of a relation or a function exists. (DOK 2)</p> <p>All.2.j. Explain the results of compositions of functions. (DOK 2)</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>

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<p>inverse of another.</p> <p>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p>		
<p>F-BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p>	<p>All.1.f. Explain and use the inverse relationship between exponential and logarithmic expressions. (DOK 2)</p>	
<p>F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</p> <p>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p>	<p>Al.3.e. Graph and analyze linear functions. (DOK 2)</p> <p>PA.2.e. Graph linear equations and non-linear equations (<math>y = x^2</math>) using multiple methods including t-tables and slope-intercept. (DOK 2)</p> <p>PA.2.f. Given a linear graph, identify its slope as positive, negative, undefined, or zero, and interpret slope as rate of change. (DOK 2)</p> <p>Al.3.b. Solve problems that involve interpreting slope as a rate of change. (DOK 2)</p> <p>Al.2.d. Explain and illustrate how a change in one variable may result in a change in another variable and apply to the relationships between independent and dependent variables. (DOK 2)</p>	<p>Specific correlations exist only by liberal interpretation of more generic MMFR objectives. The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>

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	PC.3.c. Predict the shapes of graphs of exponential, logarithmic, rational, and piece-wise functions, and verify the prediction with and without technology. (DOK 2)	
F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	AI.2.e. Graph and analyze linear functions. (DOK 2)  PC.1.b. Evaluate and apply formulas for arithmetic and geometric sequences and series. (DOK 2)	The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.
F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	AI.2.e. Graph and analyze linear functions. (DOK 2)  PC.3.c. Predict the shapes of graphs of exponential, logarithmic, rational, and piece-wise functions, and verify the prediction with and without technology. (DOK 2)	Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.
F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.	PC.2.f. Solve exponential and logarithmic equations to include real-world applications. (DOK 2)	It is possible that technology could be used in real-world application as stated in the MMFR objective.
F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.	AI.2.e. Graph and analyze linear functions. (DOK 2)	Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.
F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	T.1.a. Perform conversions across measurement systems including degree to radian measurements of angles, radian measurements to degree measurements of	Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.

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	angles, polar to rectangular coordinates, rectangular to polar coordinates, rectangular to trigonometric forms of complex numbers, and trigonometric to rectangular forms of complex numbers. (DOK 1)	
<p>F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p>	<p>T.4.c. Find exact values of trigonometric functions of special angles in the unit circle. (DOK 1) □□</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>
<p>F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosines, and tangent for <math>x</math>, <math>\pi+x</math>, and <math>2\pi-x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p>	<p>G.4.d. Explain and use the properties of 45-45-90 and 30-60-90 triangles. (DOK 2)</p> <p>T.4.d. Recognize, sketch, and interpret graphs of the six trigonometric functions and include restrictions on their domain. (DOK 2) □□</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>
<p>F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p>	<p>T.4.a. Use the unit circle to solve real-world applications and problems in mathematical settings. (DOK 3) □□</p> <p>T.4.d. Recognize, sketch, and interpret graphs of the six trigonometric functions and include restrictions on their domain. (DOK 2) □□</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>
<p>F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.□</p>	<p>T.5.b. Using graphs of functions of the form <math>f(t) = A \sin (Bt + C)</math> or <math>f(t) = A \cos (Bt + C)</math>, interpret <math>A</math>, <math>B</math>, <math>C</math> in terms of amplitude, frequency, period, and phase shift. (DOK 2)</p>	<p>The CCSS are more explicit regarding student performance expectations, whereas the MMFR are more general.</p>

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<p>F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p>		<p>There is no MMFR objective that specifically mentions graphing inverse trigonometric functions.</p>
<p>F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.□</p>	<p>T.3.d. Solve trigonometric equations in real-world situations or mathematical settings. (DOK 2)</p>	<p>There is no MMFR objective that specifically mentions inverse trigonometric functions, but it must be assumed that there are trigonometric equations that involve finding unknown angles that necessitates the use of an inverse trigonometric function.</p>
<p>F-TF.8. Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to calculate trigonometric ratios.</p>		<p>There is no MMFR objective focusing on the Pythagorean identity.</p>
<p>F-TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p>	<p>T.1.d. Explain the addition formulas for sine and cosine and use them to prove (or simplify) other trigonometric functions. (DOK 2)</p>	
	<p>T.4.b. Apply the six trigonometric functions in relation to a right triangle to solve real-world applications and problems in mathematical settings. (DOK 3)</p>	<p>This MMFR objective as stated is not found in the CCSS.</p>
	<p>All.3.b Classify functions based on sketches of their graphs. (DOK 2)</p>	<p>This MMFR objective as stated is not found in the CCSS.</p>
	<p>All.5.a. Through the use of technology, use scatter plots and linear and quadratic regression analysis to determine an appropriate function to model real-life</p>	<p>While the CCSS does not use the term regression, fitting a function to the data is found in Statistics and Probability in the</p>

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	data. (DOK 3)	MMFR.
	AA.4.a. Use technology and regression analysis to determine appropriate quadratic and cubic functions modeling real-life data. (DOK 3)	Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS Statistics and Probability.
	PC.4.a. Use regression methods available through technology to determine appropriate exponential and logarithmic functions that model real-life data. (DOK 3)	Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS Statistics and Probability.
	PC.4.b. Use regression methods available through technology to determine appropriate cubic functions that model real-life data. (DOK 3)	Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS Statistics and Probability.
	C.2.c. Determine and justify the continuity and discontinuity of functions. (DOK 2)	This MMFR objective as stated is not found in the CCSS.

## GEOMETRY (G)

CCSS Key:

- Congruence (G-CO)
- Similarity, Right Triangles, and Trigonometry (G-SRT)
- Circles (G-C)
- Expressing Geometric Properties with Equations (G-GPE)
- Geometric Measurement and Dimension (G-GMD)
- Modeling with Geometry (G-MG)
  - Modeling Standards
  - (+) Standards that Include Complex Numbers on Complex Plane In Rectangular and Polar Form (Includes Real and Imaginary Numbers)

MMFR Content Standards Key:

- Numbers and Operations (1)
- Algebra (2)
- Geometry (3)
- Measurement (4)
- Data Analysis and Probability (5)
- Depth of Knowledge (DOK)

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G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		While not necessarily focusing on knowing precise definitions, identifying and analyzing the relationships between and among points, lines, line segments, angles, and rays begins in 4 <sup>th</sup> grade MMFR. The term “perpendicular” does not appear until Algebra I MMFR. The term “parallel” first appears in Pre-Algebra MMFR.
G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs.  Compare transformations that preserve distance and angle to those that do not (e.g., translation versus	G.2.e. Determine the effects of rigid (translations, rotations, and reflections) and non-rigid (dilations) motions and compositions when performed on objects on the coordinate plane. (DOK 2)	The idea of transformations is also addressed in 4 <sup>th</sup> , 5 <sup>th</sup> , and 6 <sup>th</sup> grades MMFR.

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horizontal stretch).		
<p>G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</p>		<p>Identifying transformations and translations of two-dimensional figures begins in 4<sup>th</sup> grade MMFR, but those objectives do not call for considering the reflections and rotations that carry two-dimensional figures onto themselves.</p>
<p>G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</p>	<p>G.2.e. Determine the effects of rigid (translations, rotations, and reflections) and non-rigid (dilations) motions and compositions when performed on objects on the coordinate plane. (DOK 2)</p> <p>G.3.i. Given the pre-image or image, find figures obtained by applying reflections, translations, rotations, and dilations; describe and justify the method used. (DOK 2)</p>	<p>Inherent in the MMFR objectives is the understanding of definitions of rotations, reflections, and translations, but developing definitions regarding rotations, reflections, and translations is not stated specifically in the MMFR.</p>
<p>G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</p>	<p>G.3.i. Given the pre-image or image, find figures obtained by applying reflections, translations, rotations, and dilations; describe and justify the method used. (DOK 2)</p>	<p>While not expecting students to specify a sequence of transformations that will carry a given figure onto another as in the CCSS, the MMFR expects students to justify the method used.</p>
<p>G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</p>	<p>G.2.e. Determine the effects of rigid (translations, rotations, and reflections) and non-rigid (dilations) motions and compositions when performed on objects on the coordinate plane. (DOK 2)</p>	<p>The MMFR objective includes both rigid and non-rigid motions and compositions.</p>



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<p>G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p> <p>G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</p>	<p>G.3.e. Classify triangles and apply postulates and theorems to test for triangle inequality, congruence, and similarity. (DOK 2)</p>	<p>The MMFR objective is all-inclusive about testing triangles for inequality, congruence, and similarity but does not necessarily focus on congruence in terms of rigid motion.</p>
<p>G-CO.9. Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p>	<p>G.3.c. Identify, classify, and apply angle relationships formed by parallel lines cut by transversals. (DOK 2)</p>	<p>The CCSS expects students to prove theorems about lines and angles, while the MMFR objective calls for identifying, classifying, and applying relationships of angles formed by parallel lines cut by a transversal.</p>
<p>G-CO.10. Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p>	<p>G.3.d. Use the properties of altitudes, medians, angle bisectors, and perpendicular bisectors of triangles to solve problems. (DOK 2)</p>	<p>The contrast between the CCSS and the MMFR objective is “proving” versus “using.”</p>
<p>G-CO.11. Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i></p>		<p>Polygons of regular and irregular shapes are included in 6<sup>th</sup> grade MMFR, but the focus is drawing, labeling, and classifying, and not about proving theorems.</p>

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<p>G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></p>	<p>G.3.d. Use the properties of altitudes, medians, angle bisectors, and perpendicular bisectors of triangles to solve problems. (DOK 2)</p>	<p>The MMFR objective calls for using properties, while the CCSS expects formal geometric constructions.</p>
<p>G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</p>		<p>There is no MMFR objective with this CCSS specificity.</p>
<p>G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:</p> <ul style="list-style-type: none"> <li>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</li> <li>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</li> </ul>		<p>Use of scale factors to perform a dilation and proportional reasoning is included in 6<sup>th</sup> grade MMFR. The CCSS, however, delineates specific cases to consider.</p>

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<p>G-SRT.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p> <p>G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>	<p>G.3.e. Classify triangles and apply postulates and theorems to test for triangle inequality, congruence, and similarity. (DOK 2)</p>	<p>The CCSS focuses on similarity transformations, a term not found in the MMFR.</p>
<p>G-SRT.4. Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i></p>	<p>G.3.e. Classify triangles and apply postulates and theorems to test for triangle inequality, congruence, and similarity. (DOK 2)</p> <p>G.3.b. Develop and evaluate mathematical arguments and proofs to include paragraph, two-column, and flow chart forms. (DOK 3)</p>	<p>The CCSS is about proving theorems on triangles.</p>
<p>G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</p>	<p>G.3.e. Classify triangles and apply postulates and theorems to test for triangle inequality, congruence, and similarity. (DOK 2)</p>	<p>The CCSS is about using criteria for triangles to solve problems.</p>
<p>G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p>	<p>G.4.e. Apply the relationships of sine, cosine, and tangent to problems involving right triangles. (DOK 2)</p>	<p>The CCSS focuses on understanding similarity, while the MMFR objective is about application.</p>
<p>G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.</p>		<p>The specificity of the CCSS is not found in the MMFR.</p>

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<p>G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p>	<p>G.4.b. Solve real-world applications and mathematical problems to find missing measurements in right triangles by applying special right triangle relationships, geometric means, or trigonometric functions. (DOK 2)</p> <p>G.4.d. Explain and use the properties of 45-45-90 and 30-60-90 triangles. (DOK 2)</p> <p>G.4.e. Apply the relationships of sine, cosine, and tangent to problems involving right triangles. (DOK 2)</p> <p>G.1.c. Solve real-world or application problems that involve square roots and the Pythagorean Theorem. (DOK 3)</p>	<p>The four MMFR objectives reflect the use of trigonometric ratios and/or the Pythagorean Theorem in solving problems involving right triangles.</p>
<p>G-SRT.9. (+) Derive the formula <math>A = 1/2 ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p>		<p>Determining the area of a triangle using sine and cosine and explaining the process used is in Trigonometry MMFR.</p>
<p>G-SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>G-SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>		<p>Modeling and applying right triangle formulas, Law of Sines and Law of Cosines, to problem-solving situations is in Trigonometry MMFR.</p>
<p>G-C.1.</p>		<p>This CCSS standard is not found in</p>

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Prove that all circles are similar.		the MMFR.
<p>G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i></p>	<p>G.4.a. Use the properties of circles using arc, angle, and segment relationships to find missing measures. (DOK 2)</p>	<p>The MMFR objective focuses on finding missing measures, while the CCSS calls for identifying and describing relationships.</p>
<p>G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</p> <p>G-C.4. (+) Construct a tangent line from a point outside a given circle to the circle.</p>		<p>These two CCSS are not found in MMFR.</p>
<p>G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p>		<p>Radian measurement and sector area are in Trigonometry MMFR.</p>
<p>G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</p>	<p>G.2.b. Recognize and write the equation of a circle in standard form <math>(x-h)^2 + (y-k)^2 = r^2</math> and identify the center and radius. (DOK 2)</p>	<p>In the CCSS, students are expected to derive the equation of a circle using the Pythagorean Theorem, while the MMFR is expecting students to recognize the standard form of the equation of a circle.</p>

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<p>G-GPE.2. Derive the equation of a parabola given a focus and directrix.</p> <p>G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given foci and directrices.</p>		<p>Identifying and sketching graphs of the four conic sections: circle, parabola, ellipse, and hyperbola are included in Algebra II MMFR.</p>
<p>G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p>		<p>This CCSS was not found in the MMFR.</p>
<p>G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</p>	<p>G.2.c. Use slope to analyze and write equations for parallel and perpendicular lines. (DOK 2)</p>	<p>The MMFR objective calls for using slope, while the CCSS calls for proving the slope criteria.</p>
<p>G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</p>	<p>G.2.d. Apply the Midpoint and Distance Formulas to solve application problems involving the coordinate plane. (DOK 2)</p>	<p>The CCSS calls for a specific expectation for finding a point on a directed line segment while the MMFR calls for application of Midpoint and Distance Formulas.</p>
<p>G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</p>	<p>G.2.d. Apply the Midpoint and Distance Formulas to solve application problems involving the coordinate plane. (DOK 2)</p>	<p>Both the CCSS and the MMFR objective include using the distance formula.</p>

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<p>G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i>□</p> <p>G-GMD.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.□</p>		<p>Informal arguments about formulas using Cavalier's principle, dissection, and informal limit are not found in the MMFR.</p>
<p>G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.□</p>	<p>G.4.c. Solve real-world and mathematical problems involving the lateral area, surface area and volume of three-dimensional figures, including prisms, cylinders, cones, pyramids, and spheres. (DOK 2)</p>	<p>The MMFR objective includes lateral area and surface area of three-dimensional figures.</p>
<p>G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</p>	<p>G.3.g. Describe and draw cross-sections of prisms, cylinders, pyramids, and cones. (DOK 1)</p>	<p>The MMFR objective is not specifically defining cross sections as two-dimensional.</p>
<p>G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</p>		<p>While students do experience some fundamental preparation in 5<sup>th</sup> grade MMFR, this CCSS as stated was not found in the MMFR.</p>
<p>G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).</p>		<p>While there is an expectation that students develop and justify geometric formulas in 7<sup>th</sup> grade MMFR, there is nothing in the MMFR that addresses applying the concepts of density based on area and volume.</p>

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<p>G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).</p>		<p>This CCSS is not found in the MMFR.</p>
	<p>G.1.a. Apply problem-solving skills to solve and verify the solutions for unknown measures in similar polygons. (DOK 2)</p>	<p>Similarity of geometric figures begins in 8<sup>th</sup> CCSS.</p>
	<p>G.1.b. Given exact irrational solutions, determine the best rational estimation. (DOK 2)</p>	<p>Using rational approximations of irrational numbers is found in 8<sup>th</sup> grade CCSS.</p>
	<p>G.2.a. Represent data from geometric and real-world contexts with expressions, formulas, tables, charts, graphs, relations, and functions. (DOK 2)</p>	<p>While functions begin in 8<sup>th</sup> grade CCSS, there is no CCSS that is that inclusive.</p>
	<p>G.3.a. Use inductive reasoning to make conjectures and deductive reasoning to make valid conclusions. (DOK 3)</p>	<p>This is not explicit in the CCSS.</p>
	<p>G.3.f. Determine and justify if a given shape could be tessellated. (DOK 2)</p>	<p>Tessellations are not included in the CCSS.</p>
	<p>G.3.h. Graph a vector and determine the magnitude and direction of a given vector. (DOK 2)</p>	<p>Vectors are in the Number and Quantity CCSS.</p>
	<p>G.5.a. Apply multiple strategies and representations, including area models, to solve probability problems. (DOK 2)</p>	<p>Area models begin in 3<sup>rd</sup> grade CCSS but are not focused on solving probability problems. Probability problems are in Statistics and Probability CCSS.</p>



## Statistics and Probability

CCSS Key:

- Interpreting Categorical and Quantitative Data (S-ID)
- Making Inferences and Justifying Conclusions (S-IC)
- Conditional Probability and the Rules of Probability (S-CP)
- Using Probability to Make Decisions (S-MD)
- Modeling Standards
- (+) Standards that Include Complex Numbers on Complex Plane In Rectangular and Polar Form (Includes Real and Imaginary Numbers)

MMFR Content Standards Key:

- Numbers and Operations (1)
- Algebra (2)
- Geometry (3)
- Measurement (4)
- Data Analysis & Probability (5)
- Depth of Knowledge (DOK)

Note: For this alignment analysis, the middle column includes MMFR objectives from Statistics (S), Algebra II (AII), and Discrete Mathematics (DM).

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S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	S.3.c. Create graphs with scales that fairly display the data. (DOK 2)  S.3.b. Determine and justify the graph type that best represents a given set of data. (DOK 2)	The MMFR objective focuses on creating graphs with scales that fairly display data. Representing data involving real numbers begins in 7 <sup>th</sup> grade MMFR.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	S.4.b. Determine the most appropriate measure to describe a data set, including mean, median, mode, standard deviation, and variance. (DOK 2)  S.2.b. Calculate mean, median, mode, standard deviation, z-scores, t-scores, quartiles, and ranges, and explain their applications. (DOK 2)	Multiple sets of data are included in the CCSS.
S-ID.3. Interpret differences in shape, center, and spread	S.1.a. Describe the comparison of center and spread within	Foundations for the impact of outliers are established in grade

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<p>in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p>	<p>groups and between or across group variation. (DOK 2)</p> <p>S.2.a. Analyze and describe outliers and shape of the data including linearity and correlation across graphs and data sets. (DOK 2)</p>	<p>7, MMFR.</p>
<p>S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p>	<p>S.4.c. Use curve-fitting to make predictions from collected data. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives. Normal distribution and the normal curve are not directly addressed in the MMFR.</p>
<p>S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p>	<p>S.3.a. Organize data using graphs that are appropriate to the data set, including frequency distributions, stacked line and bar graphs, stem-and-leaf plots, scatter plot, frequency polygon, and histograms. (DOK 2)</p> <p>S.4.a. Make inferences and predictions from charts, tables, and graphs that summarize data. (DOK 3)</p>	<p>The terms “categorical data” and “two-way frequency tables” are not found in the MMFR.</p>
<p>S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</p>	<p>All.5.a. Through the use of technology, use scatter plots and linear and quadratic regression analysis to determine an appropriate function to model real-life data. (DOK 3)</p> <p>All.5.c. Model a data set using the median-fit-method with a linear equation and make predictions based on the model and the equation. (DOK 3)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>

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<p>b. Informally assess the fit of a function by plotting and analyzing residuals.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p>		
<p>S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p>	<p>S.2.d. Use algebraic concepts and methods to determine mathematical models of best fit. (DOK 2)</p>	<p>Using real-world data to express slope as a rate of change is found in Transitions to Algebra, MMFR.</p>
<p>S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.</p>	<p>All.5.c. Model a data set using the median-fit-method with a linear equation and make predictions based on the model and the equation. (DOK 3)</p> <p>S.2.a. Analyze and describe outliers and shape of the data including linearity and correlation across graphs and data sets. (DOK 2)</p>	
<p>S-ID.9. Distinguish between correlation and causation.</p>		<p>The MMFR does not directly address the expectation that students distinguish between correlation and causation.</p>
<p>S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p>	<p>S.2.c. Select and use appropriate statistical methods in decision making and hypothesis testing. (DOK 2)</p> <p>S.5.d. Compare and contrast sampling methods, including simple random sampling, stratified random sampling, and cluster sampling with regard to benefits and trade-offs. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>

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<p>S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p>	<p>S.1.g. Create simulations and experiments that correlate to theoretical probability. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>
<p>S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p>	<p>S.2.c. Select and use appropriate statistical methods in decision-making and hypothesis testing. (DOK 2)</p> <p>S.5.b. Explain the generalizability of results and types of conclusions that can be drawn from observational studies, empirical experiments, and surveys. (DOK 2)</p>	
<p>S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p>	<p>S.5.d. Compare and contrast sampling methods, including simple random sampling, stratified random sampling, and cluster sampling with regard to benefits and trade-offs. (DOK 2)</p> <p>S.4.b. Determine the most appropriate measure to describe a data set, including mean, median, mode, standard deviation, and variance. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>
<p>S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p>	<p>S.5.b. Explain the generalizability of results and types of conclusions that can be drawn from observational studies, empirical experiments, and surveys. (DOK 2)</p> <p>S.1.g. Create simulations and experiments that correlate to theoretical probability. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>

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<p>S-IC.6. Evaluate reports based on data.</p>	<p>S.5.a. Design and execute a statistical experiment, including the preparation of a report that communicates the statement of the problem, methodology, results, and conclusions. (DOK 4)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives.</p>
<p>S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p>	<p>S.1.d. Construct and interpret sample spaces, events, and tree diagrams. (DOK 2)</p> <p>DM.2.h. Add, subtract, multiply, and divide sets and find unions, intersections, differences, and complements of sets. (DOK 2)</p>	
<p>S-CP.2. Understand that two events <math>A</math> and <math>B</math> are independent if the probability of <math>A</math> and <math>B</math> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p>	<p>S.1.e. Identify types of events, including mutually exclusive, independent, and complements. (DOK 1)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives. Conditional probability is not directly addressed in the MMFR.</p>
<p>S-CP.3. Understand the conditional probability of <math>A</math> given <math>B</math> as <math>P(A \text{ and } B)/P(B)</math>, and interpret independence of <math>A</math> and <math>B</math> as saying that the conditional probability of <math>A</math> given <math>B</math> is the same as the probability of <math>A</math>, and the conditional probability of <math>B</math> given <math>A</math> is the same as the probability of <math>B</math>.</p>	<p>S.1.e. Identify types of events, including mutually exclusive, independent, and complements. (DOK 1)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives. Conditional probability is not directly addressed in the MMFR.</p>
<p>S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For</i></p>	<p>S.1.e. Identify types of events, including mutually exclusive, independent, and complements. (DOK 1)</p> <p>S.4.a. Make inferences and predictions from charts, tables,</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of MMFR or CCSS objectives. Conditional probability is not directly addressed in the</p>

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<p><i>example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p>	<p>and graphs that summarize data. (DOK 3)</p>	<p>MMFR.</p>
<p>S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p>		<p>Conditional probability, especially in the details outlined in this CCSS objective, is not directly addressed in the MMFR.</p>
<p>S-CP.6. Find the conditional probability of <math>A</math> given <math>B</math> as the fraction of <math>B</math>'s outcomes that also belong to <math>A</math>, and interpret the answer in terms of the model.</p>		<p>Conditional probability, especially in the details outlined in this CCSS objective, is not directly addressed in the MMFR.</p>
<p>S-CP.7. Apply the Addition Rule, <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math>, and interpret the answer in terms of the model.</p>		<p>Conditional probability, especially in the details outlined in this CCSS objective, is not directly addressed in the MMFR.</p>
<p>S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, <math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math>, and interpret the answer in terms of the model.</p>		<p>Conditional probability, especially in the details outlined in this CCSS objective, is not directly addressed in the MMFR.</p>
<p>SP-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve</p>	<p>S.1.c. Apply the counting principles, including permutations and combinations. (DOK 1)</p>	

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problems.		
<p>S-MD.1.                      (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.</p>	<p>S.1.i.                      Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	
<p>SP-MD.2.                      (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.</p>	<p>S.1.i.                      Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	
<p>S-MD.3.                      (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. <i>For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</i></p>	<p>S.1.g.                      Create simulations and experiments that correlate to theoretical probability. (DOK 2)</p> <p>S.1.i.                      Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.</p>
<p>S-MD.4.                      (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. <i>For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?</i></p>	<p>S.1.i.                      Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.</p>
S-MD.5.	S.1.i.	Specific correlations to CCSS

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<p>(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.</p> <p>a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.</p> <p>b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</p>	<p>Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	<p>exist only by liberal interpretation of more generic MMFR objectives.</p>
<p>S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p>	<p>S.1.i. Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.</p>
<p>S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>	<p>S.1.i. Apply the concept of a random variable to generate and interpret probability distributions. (DOK 2)</p> <p>S.2.c. Select and use appropriate statistical methods in decision-making and hypothesis testing. (DOK 2)</p>	<p>Specific correlations to CCSS exist only by liberal interpretation of more generic MMFR objectives.</p>
	<p>S.1.b. Interpret and apply the concept of the Law of Large Numbers. (DOK 2)</p>	<p>The CCSS objective expresses the initial idea that as the number of events gets infinitely larger, the experimental probability approaches the theoretical probability, which is the assertion of the Law of Large Numbers.</p>



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	S.1.f. Calculate geometric probability using two-dimensional models, and explain the processes used. (DOK 2)	This MMFR objective as stated was not found in the CCSS.
	S.1.h. Use Markov Chains to calculate probability by constructing matrix models. (DOK 2)	Although Markov Chains are associated with conditional probability distribution, this MMFR objective as stated was not found in the CCSS.
	S.4.d. Explain and defend regression models using correlation coefficients and residuals. (DOK 2)	This MMFR objective as stated was not found in the CCSS.
	S.5.c. Analyze sources of bias and sampling error(s) in studies. (DOK 3)	Although bias and sampling errors are associated with population samples, surveys, etc., the CCSS does not use those specific terms.
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